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# Modelling of mechanical behaviour of some layered soils

The problem of mechanics of periodically stratified soils which can be observed in the cases of varved clays, Miocene elays and flotation wastes are considered. The paper contains a description of layered soils and a presentation of certain homogenized model of periodic two-layered fluid-saturated porous solids based on Biot's theory of consolidation. The example important from the view point of engineering geology applications is solved.

#### INTRODUCTION

One of the most fundamental subjects in engineering geological study is the analysis of stresses, displacements and fluid flows in soils. The theory of consolidation is a common approach to these problems. The uniaxial problem of a porous fluid-saturated body was solved first by K. Terzaghi (1925, 1948), and the theory of fluid flow in porous deformable body was formulated by M. A. Biot (1941, 1962), M. A. Biot and D. G. Willis (1957). The Biot's model consisted of a homogeneous elastic matrix permeated by a network of interconnected pores filled with liquid. However, certain soils posses a stratified (layered) nonhomogeneous structure. Biot's model can be employed only to a small number of layers (for the reason of boundary conditions on the interfaces). In the case of periodically stratified soils it seems to be more suitable the applications of homogenized models in which material constants are determined in terms of the geometrical and material properties of the constituens of the bodies.

In this paper we present a certain homogenized model of periodic thin-layered fluid-saturated porous solids. The model was recently derived by S. J. Matysiak (1992) for the periodically stratified fluid-saturated porous solids in which each basic unit (fundamental layer) is composed of (n + 1) — different porous layers. Here, we apply



Fig. 1. Localization of the considered soils

1 — Miocene clays in the Carpathian Foredeep (extension of marine sediments); 2 — varved clays in the Warsaw Ice-marginal Basin; 3 — southern boundary of the Carpathian Foredeep; flotation wastes accumulated in storage ponds: 4 — ash and slag after burning of hard coal, 5 — ash and slag after burning of brown coal, 6 — postproductive wastes of the sulphur, 7 — postproductive wastes of the copper Lokalizacja analizowanych gruntów

1 — iły mioceńskie w zapadlisku przedkarpackim; 2 — iły warwowe w zastoisku warszawskim; 3 – południowa granica zapadliska przedkarpackiego; flotacyjne odpady zgromadzone w osadnikach: 4 — popioły i żużle po spaleniu wegla kamiennego, 5 — popioły i żużle po spaleniu wegla brunatnego, 6 — poprodukcyjne odpady siarkowe, 7 — poprodukcyjne odpady miedziowe

the homogenized model to the case of periodic two-layered porous solids. We present the fundamental equations of the model and solve one example important from the point of view of engineering geological applications.

The periodic or almost periodic structure can be observed in the case of Pleistocene varved clays, Miocene clays and flotation wastes (Fig. 1). The obtained model can be employed in some problems of soil mechanics (for instance to prediction of stresses, ground settlement, groundwater hydrology, analysis of wave propagation in varved clays and flotation wastes).

# DESCRIPTION OF LAYERED SOILS

#### MIOCENE CLAYS - KRAKOWIEC CLAYS

From the engineering-geological point of view, the massif of the the Miocene clays should be treated as a heterogeneous, anisotropic and discontinuous medium. The physical and mechanical properties of clays were formed during long complex geological history, when they underwent several loading and unloading cycles. The result of this is their overconsolidated state. The occurrence of the thick (hundreds-thousands meters) clay and sandy clay sediments of the Miocene is limited almost exclusively to the outer zone of the Carpathian Foredeep (Fig. 2).

The Krakowiec Clays consist of marine illitic and montmorillonitic, marly, consolidated, laminated clays of Miocene age. The content of clay fraction is between 0 and 60%. The well-developed horizontal lamination (or beddings) is the primary textural element of the clays. Megascopically, the lamination is expressed by the interstratification of predominantly dark laminae and clayey slices (with variable content of iron sulphides and organic matter), ranging in thickness from fractions of a millimetre to several centimeters, and of generally more rarely occurring sandy and silty laminae and slices light in colour and of smaller thickness (Fig. 2).

In the direction perpendicular to lamination there are two indifferently marked axis of microanisotropy intersetting approximately at right angles. On the other hand, one direction of the microanisotropy in the direction parallel to lamination (Fig. 2) can be seen. Besides, lenticles and intercalations of quartz sands, bentonites, volcanic tuffites, hard marls, mudstones as well as pyrites of marcasitic concretions, barytes and fragments of carbonificated wood occur in the Krakowiec Clays.

Microtectonic disturbances can be found in the clays in the shape of continuous and discontinuous deformations. The discontinuity surfaces present can be subdivided into surfaces of discontinuity of the joint type and of the polished type (slickensides). The weakness surfaces present in the Krakowiec Clays lead to various, but always marked decreases in their strength.

The characteristic feature of the Miocene clays is their disintegration in the outcrops into sheets and plates along of lamination surfaces. Microlaminated nature is explained by the activity of seasand climatic mechanism of sedimentation.

#### PLEISTOCENE VARVED CLAYS

Pleistocene varved clays formed in the immense lakes (sedimentary basins) which were situated before the front of a glacier. In these lakes the materials with liquefiable glacier were sedimented. The quantity of unfractional material always was not equally it have led to the stratification of the sediments. It has been stated, that in the warmer times, when the melting of ice was intensified, the flow of more thick material was observed, then the light beds were formed. On the other hand, during the colder times, the claycy material created the dark beds. In Poland (in the area of Mazovia), during Middle Polish Glaciation one of greatest lake, the thickness of varved clays change in the range 2–10 m. The complex of varved clays is composed of alternate light and dark





Fig. 3. Fragment of varved clay exposure in the brickyard (varved clays horizontally bedded); photo by E. Myslińska

Fragment ściany iłów warwowych w cegielni (iły warwowe poziomo warstwowane); fot. E. Myślińska

beds. The coarse-grained beds are lights and they are formed with the following grains: quartz, feldspars, carbonates with addition of clay minerals. The dark beds are more fine-grained and are composed of clay minerals (hydromicas) with slight admixture of kaolinite and montmorillonite, oxides, iron hydroxides with addition of the quartz (E. Myślińska, 1965).

The difference of physical and mechanical properties between the light and dark laminae is a result of specific conditions of varved clays sedimentation. Macroscopic analysis allow determine the macrostructure of the varved clays as the primary macrostructure parallelly bedded, is characterized by intercalation of light silty beds

Fig. 2. Samples of Miocene clays with horizontal (H), diagonal (D) and vertical (V) laminations Próbki iłów mioceńskich z poziomą (H), ukośną (D) i pionową (V) laminacją



Fig. 4. Exposure of industrial waste (coal ashes) in storage ponds with a visible parallelly bedded macrostructure (sometimes with diagonal microstructure) Fragment ściany popiołów węgla kamiennego zdeponowanego w osadniku z widoczną makrostrukturą równolcgle warstwowaną (miejscami z mikrostrukturą ukośną)



#### Fig. 5. Middle-cross section of layered soils

 $l_{1,1}^{\prime}$ — thickness of the layers;  $\delta$ — thickness of each fundamental unit of the body;  $\bar{\rho}$ — density of free fluid;  $\rho^{(1)}, \rho^{(2)}$ — densities of skeletons and non-free fluid of the subsequent layers;  $N^{(1)}, M^{(1)}, R^{(1)}, Q^{(1)}$  and  $N^{(2)}, M^{(2)}, R^{(2)}, Q^{(2)}$ — porous media constants of the subsequent layers (shear modulae of skeleton volumetric deformations, modulae of fluid volumetric deformations, coupling coefficients of skeletons and fluid volumetric deformations);  $b^{(1)}, b^{(2)}$ — dissipation coefficients of the subsequent layers

Schemat przekroju gruntów warstwowych

 $l_1, l_2$  — grubości warstewek;  $\delta$  — grubość warstwy podstawowej ciała;  $\bar{\rho}$  — gęstość cieczy swobodnej;  $\rho^{(1)}, \rho^{(2)}$ — gęstości szkieletów wraz z cieczą związaną poszczególnych warstewek;  $N^{(1)}, M^{(1)}, R^{(1)}, Q^{(1)}$  i  $N^{(2)}, M^{(2)}, R^{(2)}, Q^{(2)}$  — stałe materiałowe ośrodków porowałych dla poszczególnych warstewek (moduły ścinania, odkształcenia objętościowego, odkształcenia objętościowego cieczy, współczynniki wpływu odkształceń objętościowych cieczy na naprężenia w szkieletach i odwrotnie);  $b^{(1)}, b^{(2)}$  — współczynniki dyspacyjne poszczególnych warstewek

and the dark beds being more clayey. Frequently the parallelly laminated microstructure pass into diagonally laminated ones. The thickness of light beds is up to 15 cm, on the average equal some centimeters, but in general the dark beds show lower thickness from fractions of a millimetre to several centimeters (Fig. 3).

#### FLOTATION WASTES-ACCUMULATED IN STORAGE PONDS (LAGOONS)

Many postproductive (industrial) wastes for example: the ash and slag (after burning of hard coal and brown coal), the sulphur, copper wastes, etc. are deposited on the sedimentary ponds. The flotation wastes are pomped via pipelines from the plants into storages, where during the sedimentation the considerable separation and segregation of the solid particles occur. The waste matter deposited in the settling pond is represented by the deposits granulometrically corresponding to sandy-silty soils, sometimes clays. They show an aggregate structure. It is characterized by relatively low specific weight and high porosity. The ashes have both intergranular and interaggregate porosity. The contribution of particular parameters as well as the values of physical parameters are varying and depend upon the technology of burning and distance of the site of deposition from pomp outlet. Changes of granulometrical composition in the vertical and interal profile are observed. The flotation wastes are characterized by laminated microstructures. The thickness of the ashes change in the range from 0.5 to 15 cm. On the Figure 4 the alternate light and dark beds of hard coal ashes are seen.

The above presented soils are characterised by periodically or almost periodically layered structure. The description of them as homogeneous media can be rather inaccurate. Motivated by the stratified structure of the considered soils the remainder of this paper will be devoted to a certain homogenized model of periodic thin-layered fluid-saturated porous solids.

#### EQUATIONS OF THE HOMOGENIZED MODEL

Consider then nonhomogeneous body which in the natural (undeformed) configuration is composed of periodically reapeted two fluid-saturated porous elastic layers (Fig. 5). Let  $x \equiv (x_1, x_2, x_3)$  be the Cartesian coordinate system such that the axis  $x_2$  is normal to the layering. Let  $l_1, l_2$  be the thicknesses of the layers, and  $\delta$  be the thickness of each fundamental unit of the body, so  $\delta = l_1 + l_2$ . Let  $\rho$  denote the density of free fluid and  $\rho^{(1)}, \rho^{(2)}$  be the densities of skeletons and non-free fluid of the subsequent layers. By  $N^{(1)}, M^{(1)}, R^{(1)}, Q^{(1)}$  and  $N^{(2)}, M^{(2)}, R^{(2)}, Q^{(2)}$ , we denote the porous media constants. Let  $t, t \in [t_0, t_1)$  denote the time,  $u(x,t) \equiv (u_1, u_2, u_3)(x,t)$  be the displacement vector of skeleton and  $U(x,t) \equiv (U_1, U_2, U_3)(x,t)$  be the displacement vector of fluid.

According to results of the paper by S. J. Matysiak (1992) the displacements of skeleton and fluid are postulated in the form:

$$u_{i}(x_{1}, x_{2}, x_{3}, t) = w_{i}(x_{1}, x_{2}, x_{3}, t) + \underline{l(x_{2})q_{i}(x_{1}, x_{2}, x_{3}, t)},$$

$$U_{i}(x_{1}, x_{2}, x_{3}, t) = W_{i}(x_{1}, x_{2}, x_{3}, t) + \underline{l(x_{2})Q_{i}(x_{1}, x_{2}, x_{3}, t)},$$
[1]

i = 1, 2, 3,

where l(.) is the known a priori  $\delta$ -periodic function (called the shape function) and assumed to be piecewise linear:

$$l(x_2) = \begin{cases} x_2 - 0.5l_1 & \text{for } 0 \le x_2 \le l_1 \\ -\eta x_2/(1-\eta) - 0.5l_1 + l_1/(1-\eta) & \text{for } l_1 \le x_2 \le \delta \\ l(x_2 + \delta) = l(x_2), & \eta = l_1/\delta. \end{cases}$$
[2]

The functions  $w_i(.), W_i(.)$  are unknown functions interpreted as the components of maerodisplacements vectors of skeleton and fluid, respectively. The extra unknown functions  $q_i(.), Q_i(.)$  stand for the microlocal parameters for skeleton and fluid and are related with the periodic material structure of the body.

Applying on the Biot's theory of consolidation and using the homogenization procedure based on the nonstandard analysis methods given by C. Woźniak (1987),

the equations of homogenized model for the periodic two-layered fluid-saturated porous bodies take the following form (for exact explanation see, S. J. Matysiak, 1992):

$$\widetilde{N}w_{\dot{p}jj} + (\widetilde{N} + \widetilde{M} + \widetilde{Q})w_{\dot{p}ji} + (\widetilde{Q} + \widetilde{R})W_{j\gamma ji} + [N]q_{i\gamma 2} + ([M] + [Q])q_{2\gamma i} + + [N]q_{\dot{p}j}\delta_{i2} + ([Q] + [R])Q_{2\gamma i} + \widetilde{\rho}X_{i} = \widetilde{\rho}w_{i\gamma i}, \widetilde{Q}w_{\dot{p}ji} + \widetilde{R}W_{\dot{p}ji} + [Q]q_{2\gamma i} + [R]Q_{2,i} + \widetilde{\rho}X_{i} = \widetilde{\rho}W_{\dot{p}ii},$$
[3]

i, j = 1, 2, 3

and

$$q_{1} = -\frac{|N|}{\hat{N}}(w_{1'2} + w_{2'1}),$$

$$q_{2} = \frac{-2[N]\hat{R}w_{2'2} + ([Q]\hat{Q} - [M]\hat{R})w_{j'j} + ([R]\hat{Q} - [Q]\hat{R})W_{j'j}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$

$$q_{3} = -\frac{[N]}{\hat{N}}(w_{3'2} + w_{2'3}),$$

$$Q_{2} = \frac{2[N]\hat{Q}w_{2'2} + (\hat{Q}[M] - [Q]\hat{M})w_{j'j} + (\hat{Q}[Q] - \hat{M}[R])W_{j'j}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$
[4]

where:  $\widetilde{N}$ , [N],  $\widehat{N}$  — effective constants:

$$\widetilde{N} \equiv \eta N^{(1)} + (1 - \eta) N^{(2)},$$
  

$$[N] \equiv \eta (N^{(1)} - N^{(2)}),$$
  

$$\widehat{N} \equiv \eta N^{(1)} + \frac{\eta^2}{1 - \eta} N^{(2)}$$
[5]

and the effective constants  $\widetilde{M}$ , [M],  $\widehat{M}$ ,  $\widetilde{R}$ , [R],  $\widehat{R}$ ,  $\widetilde{Q}$ , [Q],  $\widehat{Q}$  are given in equation [5] by substituting for N the material constants M, R, Q.

By  $X_{i}$  i = 1, 2, 3 denote components of the body force per unit of total mass (gravitation force) and

$$\delta_{i2} = \begin{cases} 1 & \text{for } i = 2\\ 0 & \text{for } i \neq 2. \end{cases}$$
[6]

The comma in  $w_{ij}$ ,  $w_{it}$  indicates partial differentiation; so  $w_{ij} \equiv \frac{\partial w_i}{\partial x_j}$ ,  $w_{it} \equiv \frac{\partial w_i}{\partial t}$ . The summation convention holds with respect to all repeated indices. Since  $|l(x_2)| < \delta$  for every  $x_2 \in (-\infty, +\infty)$ , then for small  $\delta$  the underlined terms in equations [1] are small and will be neglected. It has be emphasized that  $l_{2}(.)$  is not small and the terms involving  $l_{2}$  cannot be neglected. So, we have:

$$u_{i} \approx w_{i}, \qquad U_{i} \approx W_{i},$$

$$u_{i\nu\alpha} \approx w_{i\nu\alpha}, \qquad u_{i\nu2} \approx w_{i\nu2} + l_{2}q_{i},$$

$$U_{i\nu\alpha} \approx W_{i\nu\alpha}, \qquad U_{i\nu2} \approx W_{i\nu2} + l_{2}Q_{i},$$

$$u_{i\nu\alpha} \approx w_{i\nu\alpha}, \qquad U_{i\nu2} \approx w_{i\nu2} + l_{2}Q_{i},$$

$$i = 1, 2, 3,$$

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where:

$$l_{2}(x_{2}) = \begin{cases} 1 & \text{for } x_{2} \in (0.l_{1}) \\ -\eta/(1-\eta) & \text{for } x_{2} \in (l_{1},\delta) \end{cases}$$
$$l_{2}(x_{2}+\delta) = l_{2}(x_{2}).$$
[8]

To determine stresses  $\sigma_{ij}^{(r)}$  and fluid pressures  $\sigma^{(r)}$  in a layer of the *r*-th kind (with material constants  $\overline{\rho}, \rho^{(r)}, N^{(r)}, M^{(r)}, R^{(r)}, Q^{(r)}; r = 1, 2$ ) we use the costitutive relations of the Biot's theory of consolidation and equations [7]. Thus, we have:

$$\sigma_{11}^{(r)} = 2N^{(r)}w_{1,1} + M^{(r)}(w_{kk} + l_{,2}q_{2}) + Q^{(r)}(W_{kk} + l_{,2}Q_{2}),$$

$$\sigma_{12}^{(r)} = N^{(r)}(w_{1,2} + w_{2,1} + l_{,2}q_{1}),$$

$$\sigma_{13}^{(r)} = N^{(r)}(w_{1,3} + w_{3,1}),$$

$$\sigma_{22}^{(r)} = 2N^{(r)}(w_{2,2} + l_{,2}q_{2}) + M^{(r)}(w_{kk} + l_{,2}q_{2}) + Q^{(r)}(W_{kk} + l_{,2}Q_{2}),$$

$$\sigma_{23}^{(r)} = N^{(r)}(w_{2,3} + w_{3,2} + l_{,2}q_{3}),$$

$$\sigma_{33}^{(r)} = 2N^{(r)}w_{3,3} + M^{(r)}(w_{kk} + l_{,2}q_{2}) + Q^{(r)}(W_{kk} + l_{,2}Q_{2}),$$

$$\sigma_{33}^{(r)} = Q^{(r)}(w_{kk} + l_{,2}q_{2}) + R^{(r)}(W_{kk} + l_{,2}Q_{2}),$$

 $r = 1, 2; \quad k = 1, 2, 3.$ 

Equations [3], [4] and [9] constitute a system of equations describing the considered periodic two-layered fluid-saturated porous body.

The microlocal parameters  $q_1, q_2, q_3, Q_2$  can be eliminated from equations [3] and [9] by using [4]. Thus, the equations of the homogenized model may be expressed in the terms of macrodisplacements  $w_i$  and  $W_i$ .



Fig. 6. Scheme of stratified layer

p — constant intensity of the boundary loading;  $K\delta$  — thickness of stratified layer, (1), (2) — numbers of layers; other explanations as in Fig. 5

Schemat warstwy laminowanej

p — stała intensywność obciążenia brzegowego;  $K\delta$  — grubość warstwy laminowancj; (1), (2) — numery warstw; pozostałe objaśnienia jak na fig. 5

## EXAMPLE

Consider a stratified fluid-saturated porous layer resting on the rigid impermeable subsoil (Fig. 6). Let  $K\delta$  be the thickness of the stratified layer, where K is a sufficiently large natural number. Let the layer be loaded by a constant force normal to the permeable boundary and the body forces are omitted. The considered problem is static and one-dimensional, so the displacement vectors of skeleton and fluid take the form:

$$u(x_2) = (0, u_2(x_2), 0),$$
  

$$U(x_2) = (0, U_2(x_2), 0).$$
[10]

Bearing in mind the above given assumptions we consider the following boundary conditions (Fig. 5):

$$\sigma_{22}^{(1)}(x_2 = 0) = P, \qquad \sigma^{(1)}(x_2 = 0) = 0,$$
$$u_2(x_2 = K\delta) = w_2(x_2 = K\delta) = 0, \qquad \sigma_{2}^{(2)}(x_2 = K\delta) = 0, \qquad [11]$$

where P is a given constant.

Using equations [10], [4], [3], [9] and boundary conditions [11] we obtain the solution of the considered problem:

$$\begin{split} u_{2}(x_{2}) &= P(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}(\beta_{2}x_{2} - \beta_{1}), & x_{2} \in B, \\ U_{2}(x_{2}) &= -P\beta_{2}K\delta(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}x_{2} + n_{2}, & x_{2} \in B, \\ \sigma_{11}^{(1)}(x_{2}) &= P\beta_{2}(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}\{M^{(1)}(1 + \gamma_{1}) + Q^{(1)}\} + \\ -P\beta_{1}(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}\{M^{(1)}\gamma_{3} + Q^{(1)}(1 + \gamma_{4})\} & x_{2} \in B, \\ \sigma_{11}^{(2)}(x_{2}) &= P\beta_{2}(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}\{M^{(2)}[1 - \eta(1 - \eta)^{-1}\gamma_{1}] - \eta(1 - \eta)^{-1}Q^{(2)}\gamma_{2}\} + \\ -P\beta_{1}(\alpha_{1}\beta_{2} - \alpha_{2}\beta_{1})^{-1}\{-\eta(1 - \eta)^{-1}M^{(2)}\gamma_{3} + Q^{(1)}[1 - \eta(1 - \eta)^{-1}\gamma_{4}]\} & x_{2} \in B, \\ \sigma_{12}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{13}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{23}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{2} \in B, \\ \sigma_{33}^{(r)}(x_{2}) &= 0, & r = 1, 2, \quad x_{3} \in B, \\ \sigma_{33}^{(r)}(x_{3}) &= 0, & r = 1, 2, \quad x_{3} \in B, \\ \sigma_{33}^{(r)}(x_{3}) &= 0, & r = 1, 2, \quad x_{3} \in B, \\ \sigma_{33}^{(r)}(x_{3}) &= 0, & r = 1, 2, \quad x_{3} \in B, \\ \sigma_{33}^{(r)}(x_{3})$$

where:

$$\gamma_{1} = \frac{[Q]\hat{Q} - 2[N]\hat{R} - [M]\hat{R}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$

$$\gamma_{2} = \frac{2[N]\hat{Q} + [M]\hat{Q} - [Q]\hat{M}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$

$$\gamma_{3} = \frac{[R]\hat{Q} - [Q]\hat{R}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$

$$\gamma_{4} = \frac{[Q]\hat{Q} - [R]\hat{M}}{\hat{M}\hat{R} - \hat{Q}^{2}},$$
[13]

$$\begin{split} \alpha_1 &= (2N^{(1)} + M^{(1)})(1 + \gamma_1) + Q^{(1)}\gamma_2, \\ \alpha_2 &= (2N^{(1)} + M^{(1)})\gamma_3 + Q^{(1)}(1 + \gamma_4), \\ \beta_1 &= Q^{(1)}(1 + \gamma_1) + R^{(1)}\gamma_2, \\ \beta_2 &= Q^{(1)}\gamma_3 + R^{(1)}(1 + \gamma_4), \qquad n_2 \in (-\infty, +\infty) \,, \end{split}$$

and  $B_i$ , i=1, 2; B denote the regions occupied by the material of the *i*-th kind and the considered stratified body, respectively.

# FINAL REMARKS

The above presented homogenized model of periodically stratified soils is characterized by the set of effective material constants given by equations [5]. To evaluate the effective constants we need to know values of the material constants of the components (layers) constituting the stratified soils and coefficient  $\eta = l_1 / \delta$  (Fig. 5). The problem of determination of the material constants for the considered clays and flotation wastes is beyond the scope of this paper.

If we assume that the skeleton is homogeneous, then the equations of the homogenized model lead to the Biot's theory of consolidation.

The above presented homogenized model of periodically stratified soils describes not only mean but also certain local variability of stresses connected with the periodic structure of the soils. The model can be applied to foundations problems on Miocene clays, varved clays and flotation wastes. Some examples of applications as well as the determination of the appropriate material constants for Pleistocene varved clays, Miocene clays and flotation wastes are the subject of our further considerations (in preparation). The initially obtained results indicate on the possibility of applications of the homogenized model into practice.

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#### MODELOWANIE MECHANICZNEGO ZACHOWANIA SIĘ PEWNYCH GRUNTÓW WARSTWOWYCH

#### Streszczenie

Artykuł dotyczy konsolidacji gruntów o periodycznej lub prawic periodycznej strukturze warstwowej. Do tego typu gruntów można zaliczyć np. iły warwowe, iły mioceńskie oraz osady poflotacyjne. Pierwsza część opracowania zawiera opis powyższych gruntów, ich własności, miejsca występowania (fig. 1–4). Ze względu na niejednorodność (strukturę warstwową), właściwe wydaje się stosowanie do opisu mechanicznego zachowania się tych gruntów pewnych modeli "zastępczych", polegających na zastąpieniu struktury warstwowej odpowiednim modelem jednorodnego ośrodka anizotropowego (tzw. modelem homogenizowanym). Model taki został wyprowadzony w drugiej części opracowania na podstawie teorii konsolidacji Biota oraz matematycznej metody homogenizacji. Pozwoliło to na podanie układu równań opisujących ruch cząsteczek szkieletu oraz cieczy. Wyznaczono parametry modelu (tzw. współczynniki efektywne) za pomocą parametrów fizycznych i grubości poszczególnych warstw. Model zilustrowano przykładem porowatej warstwy złożonej z dużej ilóści powtarzających się lamin (cienkich warstw) spoczywającej na sztywnym, nieprzepuszczalnym podłożu. Przyjęto, że warstwa jest obciążona stałymi siłami prostopadłymi do brzegu. Wyznaczono rozkłady naprężeń, ciśnienia porowego oraz przemieszczeń w dowolnym punkcie ciała.