Loading velocity in consolidation analysis

Pawel DOBAK


In consolidation CL test programming various recommendations are encountered with regard to the selection of loading velocity. The criterion of consistency of CL test results with the traditional IL system is not fully justifiable. The behaviour of soil in conditions of constant load increase shows peculiar theoretical and experimental characteristics, different from IL tests. An optimum method of proper CL test velocity is the analysis of loading velocity path in subsoil. This method allows for taking into consideration the effect of permeability decrease during increase of pore water pressure. The basic factors, scaling loading velocities in field and in laboratory conditions, are the conditions of drainage of pore water.

Paweł Dobak, Faculty of Geology, Warsaw University, Zawirki i Wigury 93, PL-02-089 Warszawa, Poland; e-mail: pdobak@geo.uj.edu.pl (received: December 14, 2001; accepted: October 24, 2002).

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INTRODUCTION

In classical consolidation theory, the issue of loading velocity is not taken into consideration. In Terzaghi’s interpretation, consolidation is a process of pore water pressure dissipation and accompanied strain at constant load. Consolidation progress in this situation depends solely upon the permeability and compressibility of soil. Observations of the soil strain process during consecutive stages of higher and higher constant load led to several conclusions with regard to the influence of the load program upon consolidation character. When increases of load were constant and small, the role of the seepage factor was reduced, while the role of creep (secondary consolidation) increased. In order to maintain the decisive role of pore water pressure dissipation in the development of the consolidation process, the system of doubling the constant value of consolidation stress during consecutive stages of loading was usually applied. Tests of consolidation under constant load are determined as the incremental loading (IL) system.

A decisive change in the approach to the loading program was brought about by implementation of CL (continuous loading) tests. This system was initially adopted with some reservations, however, it became popular due to significant shortening of the testing time and due to the fact that the results obtained provided better characterisation of the seepage in the consolidation process. In the historical progress of CL methods, three basic tendencies were emerged:
— tests at constant velocities: constant rate of loading (CRL), constant rate of strain (CRS);
— CG test with maintenance of a constant gradient of pore water pressure.

The optimum method of determination of proper loading velocity remained, however, an open question. The loading velocity in each of the test types, listed above, is limited in certain ways. In CRL tests, strains closest to the classical oedometric conditions were obtained at the relatively lowest velocity. At the same time, however, while lowering the velocity, very small pore water pressures were obtained, and the values of consolidation coefficient were too high. In CRS research, maintaining a constant rate of strain requires gradual increase of loading velocity. This is caused by the decrease of soil compressibility as stress increases. This effect is particularly clearly visible in soils that are loaded for the first time (not reconsolidated). In order to maintain a constant pore pressure gradient in CG research, adequate loading velocity increase during testing is required.
**METHODS OF DETERMINATION OF PROPER LOADING VELOCITY IN CL TESTS**

In methods of determination of loading velocity, which have been proposed since the end of the sixties, several different, but comparable criteria can be distinguished.

The first method is determination of loading velocity in such a way as to avoid exceeding the maximum permissible \( u_H/\sigma \) values, where: \( u_H \) — pore water pressure, measured at the impermeable specimen base; \( \sigma \) — total stress, applied to specimen at a given moment.

Recommendations present in Table 1 significantly varying values of \( u_H/\sigma \). Some scientists (Smith and Wahls, 1969; Gorman et al., 1978) allow for a very high \( u_H/\sigma \) ratio of 30–50%, which makes it possible to conduct the test at a high loading velocity. Wissa et al., 1971, as well as Sällfors, 1975, emphasize the fact that in order to obtain proper consolidation parameters, it is necessary to conduct the test so that \( u_H/\sigma \) does not exceed a dozen or so percent. In such cases, the loading velocity has to be adequately reduced.

The second method is to make the testing velocity dependent upon the soil type. It is known that soils, characterised by a high value of plasticity index, are much less permeable, thus the dissipation of pore water pressure takes more time. Gorman et al., 1978, analysed relations between the coefficient of consolidation \( c_v \) and plastic properties of soil, expressed by the liquid limit \( w_l \). Next, the relation between \( w_l \) and strain velocity, obtained in CG tests, was determined. Based upon these empirical correlations, Gorman et al., 1978, formulated a general recommendation, according to which the test should be conducted at velocity \( \Delta e/\Delta t \) of approximately \( 8 \cdot 10^{-3}/s \) for soils characterised by \( w_l > 60\% \), and at lower liquid limits this velocity can be doubled.

The third method of CL research velocity determination is the proposal of Lee et al., 1993, formulated in relation to conditions and the theory of great strains. A normalised dimension-less strain rate \( \beta \) was introduced and defined as:

\[
\beta = \frac{\gamma \cdot H^2}{c_v}
\]

where: \( \gamma = \Delta e/\Delta t \) — strain rate; \( H_0 \) — initial thickness of the sample; \( c_v \) — coefficient of consolidation.

Analysis of test results for marine clays from Singapore showed that for values of \( \beta < 0.1 \), a good convergence of \( c_v \) values, obtained using CL method with results of traditional determination in IL system, was achieved. This allowed for determination of the upper limit of the strain rate \( \gamma_{max} \)

\[
\gamma_{max} = \left( \frac{\Delta e}{\Delta t} \right)_{max} = \frac{0.1 \cdot c_v}{H_0^2}
\]

An advantage of the above criterion is the direct relation of test velocity to values of the coefficient \( c_v \), and thus to the seepage properties. However, determination of the level of permissible values of \( \beta \), remains questionable.

Almeida et al. (1995) tested soft-plastic estuary clays from Rio de Janeiro, and they obtained a good consistency of consolidation coefficient from IL and CRS tests, when \( \beta \) was between 0.15 and 0.28, and the ratio \( u_H/\sigma \) was between 0.18 and 0.31. However, when \( \beta \) was lowered to 0.06, and the ratio \( u_H/\sigma \) to 0.1, effects related to rheological processes of secondary consolidation were visible. According to Lee et al., 1993, reservations of this kind are not justifiable, since theoretical solutions of CRS consolidation do not include the effects of secondary consolidation.

As can be seen on the basis of this overview, a significant criterion of evaluation of consecutive methods of selection of CL research velocity is obtaining the consolidation coefficient values, consistent with the results of traditional oedometric tests. Such an approach does not take into consideration the distinctness of the consolidation process when using various methods of loading program. A detailed evaluation of methods of selection of loading velocity thus requires:

— comparison of physical conditions of the consolidation process in IL and CL test;
— determination of theoretical characteristics of consolidation for different CL test programs;
— analysis of selected experimental data;
— new recommendations for determination of the CL test velocity, depending upon geological and engineering characteristics of the problem to be solved.

**LOADING VELOCITY IN CL CONSOLIDATION THEORETICAL MODEL**

Adaptation of the classical solution of consolidation to conditions of constantly increasing load allowed for development of a model of pore pressure dissipation in CL tests (Dobak, 1999). In order to present the model calculation results in a general form, it is necessary to introduce new (Dobak, 1999) parameters:

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**Table 1**

<table>
<thead>
<tr>
<th>Recommended ( u_H/\sigma ) values</th>
<th>Soil type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 kaolinites, Ca-montmorillonites, Massena clay</td>
<td>Smith and Wahls (1969)</td>
<td></td>
</tr>
<tr>
<td>0.05 boston blue clay (artificially sedimented)</td>
<td>Wissa et al. (1971)</td>
<td></td>
</tr>
<tr>
<td>0.1–0.15 Bakebol clay</td>
<td>Sällfors (1975)</td>
<td></td>
</tr>
<tr>
<td>0.3–0.5 (( u_{h\text{max}} = 7 \text{kPa} )) silts and clays from the Coalfield of Mississippi Plains (Kentucky)</td>
<td>Gorman et al. (1978)</td>
<td></td>
</tr>
</tbody>
</table>
where:

- $H$ — current thickness of specimen equal to length of drainage in uniaxial consolidation;
- $c_v$ — coefficient of consolidation;
- $\gamma_w$ — unit weight of water;
- $k$ — coefficient of permeability;
- $M$ — modulus of one-dimensional compressibility.

— relative consolidation time $t/T(1)$

\[
t_{r=1} = \frac{H^2}{c_v} = \frac{H^2 \cdot \gamma_w}{k \cdot M}
\]  

where: $H$ — current thickness of specimen equal to length of drainage in uniaxial consolidation; $c_v$ — coefficient of consolidation; $\gamma_w$ — unit weight of water; $k$ — coefficient of permeability; $M$ — modulus of one-dimensional compressibility.

— the parameter of pore water pressure $C = \mu/\sigma$ (after Lambe and Whitman, 1969), used in mechanics of saturated soils; $C_k$ — parameter $C$ for conditions of CL test in consolidometer.

Loading programs, applied in IL and CL test, were limited for the needs of theoretical analysis to formula $\sigma = a \cdot t^n$ and characterised in Table 2.

It is worth noting that, in order to obtain CRS and CG conditions during CL tests, load increase depends upon soil compressibility and permeability, and it can be approximated by various functions (usually increasing). Model $\sigma = a \cdot t^n$, where $n > 1$ is only one example of a function of this class.

The obtained results of theoretical analysis, illustrated by Figure 1, indicate the following relationships:
- dissipation of pore water pressure during CL test depends exclusively upon the character of load increase, expressed by the exponent, and not upon a value of loading velocity.
- in the theoretical model, a single graph in the coordinate: $C_k \leftrightarrow t/t(T=1)$ corresponds with different constant rate of loading (every CRL tests described by function $\sigma = a \cdot t^n$ where $n=1$).

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**Table 2**

<table>
<thead>
<tr>
<th>Types of tests</th>
<th>Conditions of loading</th>
<th>Exponential model of stress changes $\sigma = a \cdot t^n$</th>
<th>Governing physical processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL</td>
<td>$\sigma = \text{const}$</td>
<td>$n = 0$</td>
<td>— creep of soil skeleton</td>
</tr>
<tr>
<td>CRL</td>
<td>$\Delta\sigma/\Delta t = \text{const}$</td>
<td>$n = 1$</td>
<td>— character and changes in stress increase, seepage</td>
</tr>
<tr>
<td>CRS</td>
<td>$\Delta\sigma/\Delta t$ increasing</td>
<td>$n &gt; 1$</td>
<td>— seepage</td>
</tr>
<tr>
<td>CG</td>
<td></td>
<td></td>
<td>— creep of soil skeleton</td>
</tr>
</tbody>
</table>

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**Fig. 1.** Theoretical distribution of water pressure in one-dimensional consolidation tests (after Dobak, 2000)
Similarly, the IL test system (where \( n = 0 \)) is characterised by a standardised distribution of pore water pressure. — \( \frac{u_H}{\sigma} \) values increase, when the rate of stress increases gradually during loading \((n > 1)\).

The basic formula of determination \( c_v \) from the CL test is:

\[
c_v = \frac{\Delta \sigma \cdot H^2}{\Delta t \cdot 2 \cdot u_H}
\]

where \( \Delta \sigma \) — increment of applied stress in elapsed time; \( H \) — current thickness of sample (length of seepage path); \( u_H \) — pore water pressure, measured at impermeable specimen base.

Further analysis of theoretical model shows that reliable \( c_v \) values, determined on the above basic formula are obtained for the so-called steady phase of test, that is, when \( \frac{\Delta \sigma}{\sigma} > 2 \).

It can be thus concluded that high \( \frac{u_H}{\sigma} \) values, allowed for by Smith and Wahls, 1969 and Gorman, 1978 correspond in CRL tests with the non-steady phase, when \( \frac{\Delta \sigma}{\sigma} < 2 \). Determination of reliable \( c_v \) values is difficult in this case, and it requires application of more complicated methods of interpretation (for instance, corrections, according to Janbu et al., 1970, ASTM or MR method — vide Dobak, 1999).

At the same time, rigorously low \( \frac{u_H}{\sigma} \) values, recommended by Wissa et al., 1971, leave a significant part of the steady phase outside interpretation.

As can thus be seen, the method of selection of loading velocity, discussed above, referring to postulated \( \frac{u_H}{\sigma} \) values, has significant limitations, related to CL test phases and the reliability of determination of the \( c_v \) value.

**EXPERIMENTAL EVALUATION OF CL TEST VELOCITY CRITERIA**

The theoretical model of dissipation of pore water pressure in CL tests, like the classical solution of Terzaghi, does not include the relation between compressibility and permeability of consolidated soils. Thus the comparison of test velocity criteria based upon the \( \frac{u_H}{\sigma} \) ratio with the dimensionless strain rate \( \beta \) is possible only on the basis of experimental results.

Figure 2 presents the values of the parameter \( \frac{u_H}{\sigma} \) and \( \beta \) obtained during CL tests of soils from the profile of the “Belchatów” lignite mine in central Poland.

The data presented characterise the consolidation process in the interval stress between 400 and 600 kPa. In most cases, the results obtained are in the field that meets both the condition of...
β < 0.1 and \( C_i < 0.25 \), which corresponds with steady phase conditions in CRL tests. For increasing loading velocity \( (n > 1) \), the initial part of the steady phase is situated above value \( C_i = 0.24 \). Some clay specimens, both characterised by undisturbed structure and made of soil paste, were characterised in the stress interval analysed by \( C_i \) values greater than 0.25, while at the same time meeting the condition of \( β < 0.1 \). For several analyses of Tertiary clays, the strain rate \( β \) exceeded the limit of 0.1 for \( C_i \) values between 0.15 and 0.65. This overview of experimental data shows that the criteria of test velocity, based upon parameters of the dimensionless strain rate \( β \) and the ratio \( \nu_t/\sigma \)

\[ \alpha = C_i \] are mutually independent, and in some cases meeting one of them does not automatically mean that the postulated range of values for the second has been obtained.

### INFLUENCE OF PORE WATER PRESSURE ON PERMEABILITY OF CONSOLIDATED SOIL

Detailed analyses of variability of soil consolidation characteristics, recorded during CL tests, indicate an important role of loading rate in the shaping of the consolidation process. Theoretical solutions show that at higher loading velocities, pore pressure values increase (Fig. 3). At the same time, it has been noticed that the increase of pore water pressure is usually accompanied by a sharp decrease in hydraulic conductivity (coefficient of permeability \( k \)). Comparing the results of analysis of soils with the same lithological characteristics, it can be noted that in many cases, pore water pressure can significantly modify the basic dependence of the permeability upon void ratio \( e \). Treating the size of the pore space as the most important factor of hydraulic seepage resistances, we should obtain a repeatable correlation \( \log k \leftrightarrow e \) for a given soil. However, for the same specimens in conditions of primary and secondary loading, a mutual shift of generalised \( \log k \leftrightarrow e \) graphs, characterising permeability, has been observed. For a given void ratio, lower coefficient \( k \) values are obtained during primary loading, and higher values are obtained during secondary loading. This was directly connected with obtaining higher values of pore water pressure during primary loading.

A similar shift of \( \log k \leftrightarrow e \) graphs has been observed in results of tests conducted at various stress velocities \( \Delta\sigma/\Delta t \). For the comparable values of void ratio (or axial strain), greater pore water pressure values are obtained when applying higher \( \Delta\sigma/\Delta t \). This effect caused a decrease in \( k \) values, calculated on the basis of consolidation process.

As a consequence, a mutual shift of \( \log k \leftrightarrow e \) graphs, corresponding with different velocities of CL tests, takes place. Such effects have been observed during consolidation of dark Tertiary clays from the “Belchatów” mine (Fig. 4), some specimens of green clays and Odra tills. The shift of \( \log k \leftrightarrow e \) graphs obtained for different CL test velocities has also been noted in a monograph prepared on the Bothkennar Clay (Nash et al., 1992). These correlations, although they do not occur in the process of all analyses conducted, may point to a specific role and influence of the pore water pressure values in the pore space upon the shaping of medium effective permeability.

### METHOD OF ANALYSIS OF LOADING VELOCITY PATH IN CL TEST PROGRAMMING

It seems, on the basis of the presented theoretical and experimental correlations, that parameters of soil permeability are variable and dependent upon the loading velocity path. Just as appropriate determination of soil compressibility requires analysis of the stress path, obtaining reliable consolidation parameters requires analysis of the expected velocity of stress changes in the field. This pertains to various areas of engineering activity: consolidation as a result of change in stress during drainage of the soil mass, settlement under a constructed building or a waste dump, etc. The proposed new method of determination of appropriate velocity during laboratory tests is based upon the rule of a model similarity with the consolidation process for soil mass conditions.

Good consistency of consolidation parameters, obtained in CL tests, with theoretical solutions, allows for usage of the \( c_i \) calculation formula to determine the relationship between rate of loading and the rate of change of stresses in the soil mass (Dobak, 1995).

We assume that \( c_i \) values express a set of certain physical properties of the soil and are the same in a homogeneous soil layer and in a laboratory specimen that represents it: \( c_i \) _lab = \( c_i \) _mass
The formula of determination of $c_v$ during constant loading increase results in the relation:

$$\frac{\Delta \sigma}{\Delta t} \cdot \frac{H^2_{\text{lab}}}{2 \cdot (u_{\text{H}})_{\text{lab}}} = \frac{\Delta \sigma}{\Delta t} \cdot \frac{H^2_{\text{mass}}}{2 \cdot (u_{\text{H}})_{\text{mass}}}$$

where: index “lab” — shows parameters obtaining in laboratory conditions, index “mass” — marked the condition existing in the soil mass.

Transforming the above relation, we obtain:

$$\frac{\Delta \sigma}{\Delta t} \cdot \frac{\Delta \sigma}{\Delta t} = \frac{H^2_{\text{mass}}}{H^2_{\text{lab}}} \cdot \frac{(u_{\text{H}})_{\text{lab}}}{(u_{\text{H}})_{\text{mass}}}$$

A factor scaling the loading increase velocity in laboratory conditions in relation to conditions existing in the soil mass is thus the following expression:

$$\frac{H^2_{\text{mass}}}{H^2_{\text{lab}}} \cdot \frac{(u_{\text{H}})_{\text{lab}}}{(u_{\text{H}})_{\text{mass}}}$$

The drainage length is assumed to be as follows:

- in laboratory tests — the thickness of the specimen (in one dimensional drainage from the specimen bottom to the top);
- in the soil mass — half of the impermeable layer thickness, limited at the top and bottom by permeable strata (two-way drainage).

Such an assumption corresponds with the situation where the surplus of pore water pressure is quickly dissipated in the

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**Fig. 4.** The effect of decrease of permeability in consolidated soil connected with increase in pore water pressure

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**Fig. 5.** Nomogram for determination of CL test velocity (after Dobak, 1999)
permeable stratum, while in the slowly consolidated impermeable layer it is maintained over a much longer period of time. Additional evaluation of variability of geological structure with regard to permeable properties of soils is necessary here. Existence of various beds of permeable soil and privileged filtration paths along the factures can significantly improve the drainage conditions in relation to the model solution, assuming one-axial (top–bottom) seepage within the stratum.

Adequate determination of test velocities depends also upon divergences between adequate pore water pressures in laboratory test and in the soil mass. The results of field observation (Kebbaj et al., 1988; Leroueil et al., 1992) point out the similarity of distribution of pore water pressure in normally consolidated formations and in laboratory specimens. For such conditions, the following ratio \( (s_u) \) can be assumed:

\[
s_u = \frac{(u_H)_{lab}}{(u_H)_{mass}} = 1 \tag{8}
\]

In older, reconsolidated formations, induction of pore water pressures in the soil mass may encounter resistances from the compacted soil fabric. Assessment of testing velocity on the basis of a formula \([6]\) leads in such case to an increase of the scaling factor proportionally to the ratio \((u_H)_{lab} / (u_H)_{mass}\).

Deviations of \(s_u\) from value 1, assumed in the model, are, after adequate transformations of the formula \([6]\), equivalent to the change of drainage path in massif \(\Delta H_{mass}\), replaced by coefficient :

\[
\zeta = \frac{H_{mass} + \Delta H_{mass}}{H_{mass}} \tag{9}
\]

then:

\[
H_{mass}^2 \cdot s_u = (\zeta \cdot H_{mass})^2 \tag{10}
\]

thus:

\[
\zeta = (s_u)^{0.5} \tag{11}
\]

Determination of loading velocity in CL test in relation to conditions of stress changes in the field is illustrated by a nomogram (Fig. 5). It was designed on the basis of the following assumptions:

— drainage length in laboratory test is equal to 2 cm;
— effective drainage length in soil mass is determined on the basis of the relation:

\[
H_{mass} = \left( \frac{(u_H)_{lab}}{(u_H)_{mass}} \right)^{0.5} \cdot H_{mass} = \zeta \cdot H_{mass} \tag{12}
\]

Then, for the corrected \(H'_{mass}\) value and the actual (or expected) stress increase velocity in the field, the recommended loading increase velocity in the CL test should be read on the monogram.

The monogram scales have been adapted to units used most often in laboratory practice (kPa/h) and in field analyses of changes of stress:

— in conditions of field drainage: depression of water level in year (\(= 3.2 \times 10^{-5}\) Pa/s);
— for load increase caused by objects: kPa / year (\(= 3.2 \times 10^{-5}\) Pa/s).
Through adequate transformation of the above monogram, it is possible to obtain 3D representation (Fig. 6), where:
 — on the x axis — soil series loading velocities in the soil mass are described: $(\Delta \sigma/\Delta t)_{\text{mass}} \text{ kPa/a} $;
 — on the y axis — drainage lengths for analysed layers are described (m);
 — coordinate z — corresponds with the values of stress application velocity, most often encountered in laboratory research $(\Delta \sigma/\Delta t)_{\text{lab}} \text{ MPa/h}$ and it can be presented in 2D as an isoline.

Such representation allows, for example, for pointing out areas corresponding with conditions of stress increase in a drained soil mass, or in the foundations of a heavy engineering object.

**CONCLUSIONS**

1. Criteria of proper loading velocity in CL tests, applied so far, are not adapted to limitations resulting from theoretical solutions for the consolidation process. When allowing for CRL test velocities, during which the pore water pressure in transmission is greater than 24% of current load, a non-steady phase of test occurs. In such cases, $c_v$ values should be determined, using correction procedures (such as Janbu’s correction or the MR method). In CRS and CG tests, exceeding the relative consolidation time $t/(1+t) > 2$ is the criterion of reaching the steady phase.

2. Water pressure in pores of consolidated soil depends upon rate of stress, and it significantly influences the changes in permeability conditions. For the same porosities at higher pore pressures, lower filtration coefficient values are obtained. This indicates an important role of loading velocity in governing the consolidation process.

3. To represent the variable consolidation conditions in the soil mass, it is necessary to conduct CL research at velocity adapted to loading velocity path in given geological and engineering conditions (field drainage, foundation load by an object, dumping ground etc.).

4. The proposed method of determination of CL test velocity on the basis of analysis of load velocity path in the soil mass allows for taking into consideration the influence of variable permeability conditions upon the consolidation process.

**REFERENCES**


ASTM D 4186-89 — Standard test method for one dimensional consolidation properties of soils using controlled — strain loading.


