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Evaluation of soil permeability from consolidation analysis based on Terzaghi's and Biot's theories

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Proper evaluation of permeability parameters has a crucial signification in multi-scale prediction of consolidation. Drainage path is connected with the time of one-dimensional consolidation in Terzaghi's theory and numerous later solutions. That is why the evaluation of permeability factor allows to model the settlement's time of geological layers based on results obtained on a laboratory scale. Numerous tests show a difference of the consolidation degree obtained from one-dimensional strain course and pore pressure distribution. Evaluation of consolidation coefficient c_v based on a newly proposed method allows better understanding of the basic reasons of unconventional behaviour of tested soils. Unparallel characteristics of strain and pore pressure distribution show important role of soil's skeleton creep and its relation to permeability aspects. Solutions proposed by Biot's theory allow analysing the different velocity of strain and pore pressure distribution. The article presents a new approximated method based on Terzaghi's theory and evaluation of parameters which are necessary in application of Biot's one-dimensional solution. These parameters are connected with compressibility and coefficient of permeability. The goal of presented methodological studies is comparison between permeability obtained from application of Terzaghi's and Biot's theories. It allows trying to explain physical reasons of differences between applied models and testing results, and improving the methodology of one-dimensional consolidations tests.

Key words: consolidation, cohesive soils, Terzaghi's theory, Biot's theory, permeability, geological barriers, insulation properties.

INTRODUCTION

Determining permeability of weakly permeable soils as a result of laboratory testing is an important issue in assessing insulation properties of geological barriers of different thicknesses (Garbulewski, 2000; Majer, 2005), and for settlements prediction. Establishing permeability factor in cohesive soils is connected with many difficulties of both technical and interpretational nature (Kaczyński et al., 2000). It concerns, among others, non-linear relation between permeability rate and hydraulic gradient in pre-linear phase of the process (Kovacs, 1981), duration of analysis and related estimation errors.

An indirect method of permeability factor determination in laboratory tests is the consolidation process analysis (Pane et al., 1983). According to Terzaghi's theory (1925) and Biot's theory (1941), permeability factor is a driving factor for consolidation. Water flow in consolidated, porous soil has a character of source field, in which:

$$div\bar{v} \quad 0 \qquad [1]$$

and flow sources may theoretically occur in each point of consolidated layer (Glazer, 1985).

The main advantage and continuous vitality of consolidation theory developed by Terzaghi (1925) is the ability to graduate the process in relation to the soil permeability and the drainage path length. It allows to model the issue in different geological conditions. The solution proposed by Terzaghi (1925) is based on substantial simplifications which were corrected gradually starting with the work of Schiffman (1958), with regard to different boundary conditions: the load increase, the length of drainage path, large and small deformations etc. These numerous scenarios lead to optimal methods of experimental data interpretation (Gibson, 1981; Zindarcic et al., 1983). It should be noted that the criterion for the selection of the proper way to assess the conditions of the consolidation process in each case is a preliminary assessment of the level of compliance (or divergence) of soil behavior in relation to the standard theoretical solutions. It will be presented below.

Implementation of Biot's model (1941) for solving practical tasks results from generalization of the consolidation process in relation to the assumptions made by Terzaghi (1925) and his successors. It has its basis in the physical premises which take into consideration the ability to describe the initial deformations of the consolidated soil at the time of the load is applied, and it was not applied in Terzaghi's model (1925). Mathematical description of the deformation state in consolidometer, adopted in the study, is a simple reduction of the full set of equations of Biot's theory (1941) to the uniaxial issue.

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THEORETICAL SOLUTIONS

EVALUATION OF PERMEABILITY-CONSOLIDATION PARAMETERS ON THE BASIS OF TERZAGHI'S MODEL

Coefficient of permeability k, based on Terzaghi's theory (1925), is defined by the fundamental formula:

$$k = \frac{\gamma_w \, c_v}{M_0} \qquad [2]$$

where: c_v – coefficient of one-dimensional (vertical) consolidation [m^2/s]; γ_w – unit weight of water; M_0 – oedometric modulus [kPa]

Defining these parameters depends on pre-set assumptions, however, consolidation coefficient c_v causes major interpretational difficulties.

In differential equation for consolidation, according to Terzaghi's theory, c_v is a constant value describing phase space of admissible solutions. However, input assumptions made in formulating this theory are difficult to meet in practice. Thus, as the process progressed, numerous experimental nonconformities were noted in relation to theoretical solutions (Duncan, 1993; Dobak, 1999). Other noted observations were: variety of c_v results dependent on the interpretation method, and further development of permeability-consolidation theory by changes in boundary conditions (Poskitt, 1969; Mikasa and Takada, 1986).

Still, the assessment of conformity between the theoretical and experimental characteristics remains a basic problem (Dobak and Pająk, 2011).

Previously applied methods of permeability factor calculation, based on 50 and 90% consolidation progress points (Taylor, 1948) were substituted by the analysis of the whole experimental curve or its most important part (Parkin, 1978).

The article describes the comparison method of experimental relations and a family of theoretical curves for chosen consolidation coefficient values.

The results concerned are uniaxial consolidation of IL type with registration of axial strain ε_i and pore pressure u_i in changing time t_i from the beginning of the load application.

The following values are determined:

Degree of consolidation $U_{\varepsilon,i}$ on the basis of sample settlement progress:

$$U_{\varepsilon,i} = \frac{h_0 - h_i}{h_0 - h_f} \qquad [3]$$

where: h_0 – initial height of sample at analysed step of loading; h_i – height of sample at any time after application of analysed step of loading; h_f – final height of sample at analysed step of loading.

Degree of consolidation $U_{u,i}$ based on pore pressure dissipation:

$$U_{u,i} = \frac{u_0 - u_i}{u_0 - u_f} \qquad [4]$$

where: u_0 – initial pore water pressure (maximal value) after application of analysed step of loading; u_i – pore water pressure at any time after application of analysed step of loading; u_f – final pore water pressure at analysed step of loading.

So obtained values $U_{\varepsilon,i}$ and $U_{u,i}$ are usually different for the same time t_i due to the fact that pore pressure dissipation u is in practice hardly compliant with uniaxial strain ε and porous space decrease. This diversification is an important indicator of the nature of the process, where contribution of filtration and solid particles creeping conditions change. It is the consequence of real complex character of consolidation process (Yoshikuni et al., 1995).

Comparison of strain and pore pressure dissipation progress with selected theoretical distributions of consolidation process is made independently for experimental characteristics $U_{\varepsilon,i}$–t_i and $U_{u,i}$–t_i.

To determine the set of theoretical relations $U_{n,i}$*–t_i, n-number of similar values of consolidation coefficient $c_{v,n}$* is assumed. The values are chosen in relation to the obtained experimental process path.

Assumed changes of optional consolidation coefficient $c_{v,n}$* values are determined by m factor:

$$c_{v,n+1}^* - c_{v,n}^* = m\, c_{v,n}^* \qquad [5]$$

For each t_i, the following values were calculated:

– non-dimensional time coefficient $T_{n,i}$*:

$$T_{n,i} = \frac{c_{v,n}^* \cdot t_i}{h_i^2} \qquad [6]$$

where: t_i – any time after application of analysed step of loading; h_i – height of sample corresponding to t_i value;

– consolidation degree $U_{n,i}$* (based on T–U relations; Taylor, 1948) corresponding to rectangular distribution of pore pressure excess in the axis of consolidated layer).

Theoretical distributions $U_{n,i}$*–t_i are compared with experimental characteristics $U_{\varepsilon,i}$–t_i and $U_{u,i}$–t_i.

For this purpose, differences between experimental values of consolidation degree $U_{\varepsilon,i}$, $U_{u,i}$ and $U_{n,i}$*, calculated at given optional c_{v,n^*} values for each considered t_i value, were determined.

As a criterion for compliance of theoretical and experimental characteristics, weighted average values of the above-mentioned differences were taken to calculations, described by the following formulas:

$$d_{n,\varepsilon} = \frac{\sum \frac{|U_{\varepsilon,i} - U_{n,i}^*|}{U_{\varepsilon,i}} w_{n,i}}{\sum w_{n,i}} \qquad [7]$$

$$d_{n,u} = \frac{\sum \frac{|U_{u,i} - U_{n,i}^*|}{U_{u,i}} w_{n,i}}{\sum w_{n,i}} \qquad [8]$$

Where the weighted average stands as a range area in the surrounding of each $U_{n,i}$* point, according to the formula:

$$w_{n,i} = \frac{U_{n,i}^* - U_{n,i-1}^*}{2} + \frac{U_{n,i+1} - U_{n,i}^*}{2} \qquad [9]$$

Values $d_{n,\varepsilon}$, $d_{n,u}$ obtained from comparison of experimental curves with theoretically assumed ones for consecutive consolidation coefficients $c_{v,n}$* are compiled on $d_{n,\varepsilon}$–$c_{v,n}$*, $d_{n,u}$–$c_{v,n}$*

graphs. Minimal d_n values point to this $c_{v,n}$ consolidation coefficient value, which is most proximal to experimental distribution.

Values d_n at the same time stand for the qualitative proximity level index of theoretical characteristics to the experimental results. The smaller d_n is, the more accurate adjustment of theoretical curve to the experimental one is.

Determination and comparison of c_v values, being optimally proximal to theoretical distributions, allows also for quantitative influence evaluation of permeability and strain in consolidation process. Theoretically, it is assumed that if the pore pressure dissipation is a ruling factor for consolidation process, then U characteristics obtained on the basis of the settlement progress should be equal to non-dimensional characteristics of pore pressure dissipation in time. Also the values of consolidation coefficient $c_{v(\varepsilon)}$ and $c_{v(u)}$ as well as coefficient of permeability $k_{(\varepsilon)}$ and $k_{(u)}$, determined respectively on the basis of sample settlement analysis and pore pressure measurements, should be the same. In fact, deviations from the model are observed in laboratory tests.

Quantitatively, these discrepancies can be determined by introducing δ factor for values of c_v and k obtained for each step of loading:

$$\delta = \frac{c_{v(\varepsilon)} - c_{v(u)}}{c_{v(u)}} = \frac{k_{(\varepsilon)} - k_{(u)}}{k_{(u)}} \quad [10]$$

Value $\delta = 0$ means full, model compliance of consolidation progress characteristics analysed on the basis of the strain and pore pressure dissipation.

Positive values of δ factor point to the delay of pore pressure dissipation process in relation to the strain progress. This results from flow choking, connected both with structural and hydrodynamic factors.

Negative values of δ factor indicate an easier pore pressure dissipation, which can be caused by transformation of the soil structure during consolidation and optional favourable filtration pathways created after dissipation of local excess of pore water pressure or, also possible, leakages of testing equipment.

EVALUATION OF PERMEABILITY-CONSOLIDATION PARAMETERS ON THE BASIS OF BIOT'S MODEL

The basis for determining permeability factor is the analysis of Biot's consolidation theory equations in uniaxial strain state. Such a state is performed in consolidometric IL tests. Construction of solution was given by Gaszyński (1984). In the further course of analysis, the following formulas will be used: consolidating soil layer margin settlement $w(t)$ and pore pressure dissipation $\sigma(t)$ at its non-permeable margin. The formulas for h soil thickness and q consolidation load are:

$$w(t) = \frac{hq}{M+2N}\left[1 - F(t)\right] + \frac{hqR}{R(M+2N) - H^2}F(t) \quad [11]$$

$$\sigma(t) = \frac{qRH}{R(M+2N) - H^2}F(t) \quad [12]$$

where the function $F(t)$ equals:

$$F(t) = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left[-\pi^2(2n+1)^2\frac{kB}{4h^2}t\right] \quad [13]$$

Coefficients: A, N, Q, R ([Pa]) and k ([m^4/(Ns)]) are Biot's parameters. The remaining ones stay related as below:

$$M = A - \frac{Q^2}{R} \quad [14]_1$$

$$H = Q + R \quad [14]_2$$

$$B = \frac{R^2(M+2N)}{R(M+2N) - H^2} \quad [14]_3$$

The function $F(t)$ has the following properties:

$$\lim_{t\to 0}F(t) = 1, \quad \lim_{n\to\infty}F(t) = 0 \quad [15]$$

thus we obtain:

$$w(t\to 0) = w_0 = \frac{hqR}{R(M+2n) - H^2} = \frac{hq}{E_p} \quad [16]$$

$$w(t\to\infty) = w_k = \frac{hq}{M+2N} = \frac{hq}{E_k} \quad [17]$$

The introduced values E_k and E_p bring an interesting and important physical sense:

– characterize soil compressibility at the end of the consolidation process, thus it is module of final compressibility:

$$E_k = M + 2N \quad [18]$$

– characterize soil compressibility at the beginning of the consolidation process, thus it is module of initial compressibility:

$$E_p = \frac{R(M+2N) - H^2}{R} \quad [19]$$

From the conditions described above, an obvious relation comes out:

$$E_p = E_k - \frac{H^2}{R} \quad [20]$$

Therefore, in the light of introduced markings, it is profitable to establish the following formulas for soil settlement and pore pressure dissipation:

$$w(t) = \frac{hq}{E_k}\left[1 - F(t)\right] + \frac{hq}{E_p}F(t) \quad [21]$$

$$\sigma(t) = q\frac{H}{E_p}F(t) \quad [22]$$

Using given elements of the above-mentioned solution, the following values are determined: E_p, E_k, R, H and k. It is not equivocal with determination of the basic Biot's model parameters given earlier (as given in Gaszyński, 1984), however, stands sufficient for the description of permeability coefficient.

The next stages of parameters identification are:

1. The settlement measured at the initial moment is extracted from consolidation curve. It is assumed as w_0 value, which occurs in the outcome of initial-edge consolidation model solution – w_0. Procedure for establishing of the initial consolidation moment is described in the test methodology. Using this parameter, the initial compressibility modulus E_p can be derived from the formula [16]: $E_p = hq/w_0$.

2. Settlement given for the final sample height is extracted from the consolidation curve. It is assumed as previously, that this value occurs in the initial-boundary consolidation model solution – w_k. Using this parameter, the final consolidation modulus E_k can be derived from the formula [17]: $E_k = hq/w_k$.

3. The registered value of pore pressure at the initial moment – $_0$ is put into the formula [22]. Thus, it is calculated as follows: $H = {}_0E_p/q$.

4. From the formula [20], the following value is obtained:

$$R \quad \frac{H^2}{E_p \quad E_k}$$

5. Using already known parameters, from the formula [14]3, it gives: $B = R\,E_k/E_p$.

At this point, almost all parameters necessary for the settlement equation [21] of the sample in the consolidometer are available. The only remaining parameter to obtain is coefficient of permeability k.

6. Determination of coefficient of permeability k is done through inscription of theoretical solution function into the experiment results, using an optimization method – error reduction. One of the options is described in the previous chapter.

As it was already mentioned, the given solution does not lead to the full identification of parameters from Biot's model. It results from the range of measurements carried out during consolidation test described below.

METHODOLOGY OF LABORATORY TESTS

All tested soils are from Chmielów (Poland). They are of Miocene age and represent the Krakowiec Clays deposited in the Fore-Carpathian Basin. Their physical parameters are as follows:

– fraction content: clay – 42%, silt – 44%, sand – 14%;
– density of solid particles $_s$ = 2.72 g/cm^3, plastic limit w_p = 24.6% liquid limit w_L= 65.0%.

The parameters of soil paste samples of the Krakowiec Clays prepared for the test are given in Table 1.

These results from the measurements carried out during consolidation test are described below.

RESEARCH ASSUMPTIONS

Soil-paste samples of the Krakowiec Clays were tested in a uniaxial consolidometer, in a ring with 75.0 mm diameter and the initial height of about 29–29.5 mm.

The program assumes consolidation tests under effective stress values: 300, 600, 900 and 1000 kPa. The tests were made in a chamber of maximum allowed pressure 1100 kPa. The back pressure is established at the level of 90 kPa. Applied external loads are: 390, 690, 990 and 1090 kPa.

DESCRIPTION OF TEST EQUIPMENT

External load is applied to the top of the sample through hydrostatic pressure generated by the water placed in an isolated rubber membrane. Back pressure is given to the top of the sample through a pipe crosscutting the membrane. Subsequently, the membrane is placed on a porous filter disc. Pore pressure is measured at the bottom of the sample (Fig. 1).

Before installing the sample, all watering pipes are de-aired. After installing the sample, covering with the membrane and mounting all together, the water is introduced into the membrane while the vent 1 is opened.

Water is delivered to the space between the membrane and sample through the back pressure pipe, while the vent 2 stays open. All pressure and height tensors shall be now reset.

RESEARCH STAGES

I. Saturation. Saturation of porous space with water was led in stages with subsequent external pressure values: 100, 150, 200, 250, 300 kPa, at respective back pressure values: 90, 140, 190, 240, 290 kPa. Saturation was finished at the following parameters: external pressure of 300 kPa, back pressure 290 kPa, and pore pressure stabilized at 286 kPa. The initial sample saturation reached 97%.

II. Consolidation tests schemes. Consolidation process progress, in particular its initiation, significantly depends on the level of applied load.

During testing, the sample should be treated with external and back pressures. Various options are possible:

A. Simultaneous opening of both external and back pressure valves. As a result of this set of loads, the pore pressure in the sample increases at the beginning, the sample is drained and then settles. After the pore pressure increases at the initial phase, it decreases to the value close to the back pressure.

B. Opening of the external pressure valve with the back pressure valve closed. Pore pressure increases and stabilizes at certain level; the sample does not settle. After the pore pressure stabilizes, the back pressure valve is opened. The pore pressure decreases to the value of

Table 1

Moisture and density changes of Krakowiec Clays soil-paste samples

Sample no.	Introductory consolidation of soil-paste: moisture w		IL consolidation test		
			initial state (before first step of loading)	final state (after 4th step of loading)	
	initial w_i [%]	final $w_f = w_0$ [%]	$_0$ [g/cm^3]	w_4 [%]	$_4$ [g/cm^3]
IK1	62.0	36.1	1.87	24.4	2.03
IK2	63.6	36.4	1.80	23.9	2.04
IK3	61.4	30.1	1.90	23.0	1.96

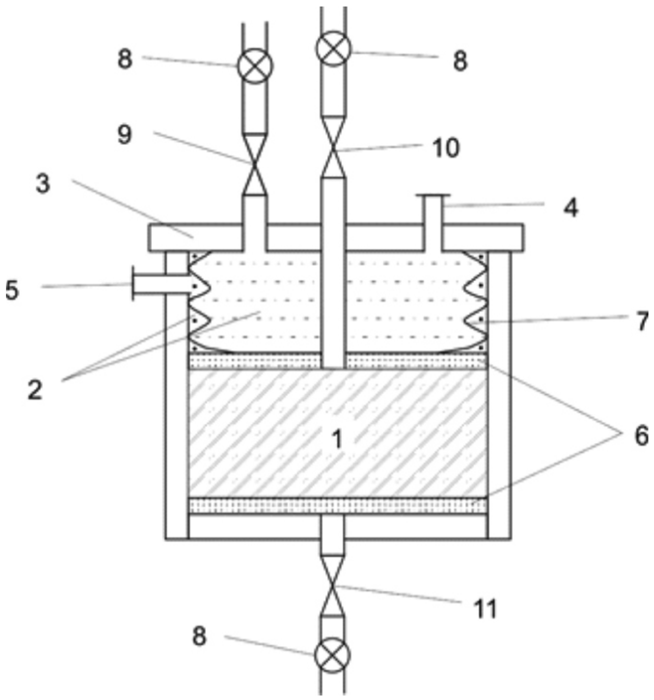


Fig. 1 The set of loading and monitoring system for soil behavior during consolidometer testg

1 – soil sample, 2 – water, 3 – cover, 4 – vent 1, 5 – vent 2, 6 – porous filter disc, 7 – rubber membrane, 8 – valve, 9 – the external pressure tensor, 10 – back pressure tensor, 11 – pore pressure tensor

back pressure with simultaneous settlement of the sample.

III. IL consolidation respectively to the subsequent load levels. For all stages of external stress $_z$ (390, 690, 990 and 1090 kPa), all the tests were conducted according to the option B (above).

The back pressure u_w was decreased to the level of ca. 90 kPa. Pore pressure u increased from the level respective for saturation end to $u_{\max}$ value, which was the basis for determination of initial pressure value $u_0 = u_{\max} - u_w$ on the given load level $_n = \ _z - \ _w$. Opening of the back pressure valve started the consolidation process and, from this point, the tendency towards equality between pore pressure u and back pressure at the level of ca. 90 kPa was observed together with sample settlement declining.

RESULTS

Both homogenization of the analysed soil and the accurate preparation procedure with consecutive saturation and initial consolidation, allowed obtaining the settlement progress and pore pressure characteristics very close to each other.

At subsequent load stages, compressibility modulus increases progressively, which is illustrated by multidimensional relation $M_0 = 0.02 \ _n^2 - 1.23 \ _n + 3485.1$, where $R^2 = 0.977$.

In the $_n$ range from 300 to 1000 kPa with more than a threefold load increase, five times greater compressibility modulus was obtained with a faster stiffness increase noted at higher load stages. This simple characteristic reflects structural changes caused in soil pasta by consecutive consolidation.

As a consequence of structural changes, pore pressure participates in load transfer, both at the initial consolidation stage (immediately after subsequent load application) and during the whole process of pore pressure dissipation in real time. Analysing pore pressure value $C_{IL} = u/ \ _n$, significant decrease of the initial value C_{IL0} at subsequent load stages can be noted.

Also, significant diversification of C_{IL0} in the samples was observed at the step $_n$ = 300 kPa. The highest C_{IL} values close to model value = 1 were noted at the initial 300 kPa load of sample IK1. Successive C_{IL0} decrease is observed at subsequent load stages (Fig. 2).

It results from the successive soil stiffness increase and decreasing porosity.

Relatively high differentiation of C_{IL0} values was obtained in simultaneously tested samples at the same load. It indicates individual liquid phase reaction under conditions of incremental loading.

However, the analysis of C_{IL} changes in time reveals close and recurrent dependence of pore pressure dissipation on the load value in different samples. It indicates a trend of consolidation – permeability changes and is a physical illustration of permeability decrease in consolidated soil medium.

INTERPRETATION

Transition from physical description to comparisons related to application of Terzaghi's theory allows distinguishing separate medium behaviour. It is relatively closely related to subsequent consolidation levels. Characterization of consolidation behaviour independently, by the observation of axial strain as well as pore pressure progress, allows distinguishing three models of behaviour.

I – where pore pressure dissipation is delayed in relation to the progress of sample settlement. These conditions correspond to factor value > 0 (in the course of analysis, ranges from 0.2 to 4.5). This situation indicates intensive rebuild of solid particles and occurs at the initial load stages (this case is shown in Figure 3).

II – where pore pressure dissipation is significantly consistent with sample settlement in time. This scenario reflects general assumptions of consolidation-permeability theory. Therefore, it may stand as a prerequisite for reliability assessment of the obtained consolidation-permeability parameters. Factor is thus proximal to 0 value.

III – where pore pressure dissipation is faster than the settlement and factor < 0. Such situation occurred in IK2 sample

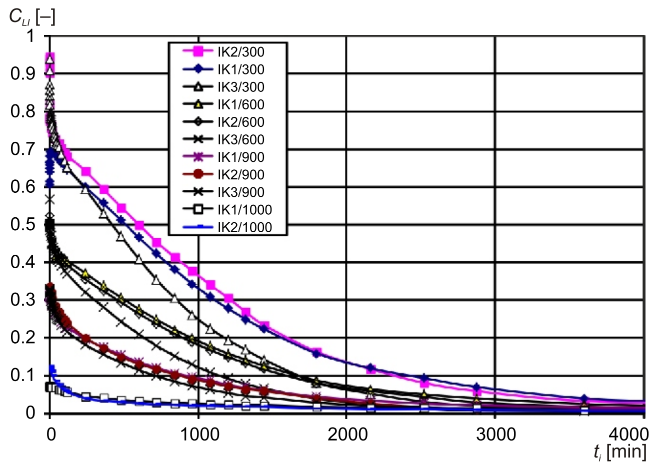


Fig. 2. Changes of pore water pressure parameters (C_{IL} and C_{IL0}) during IL tests

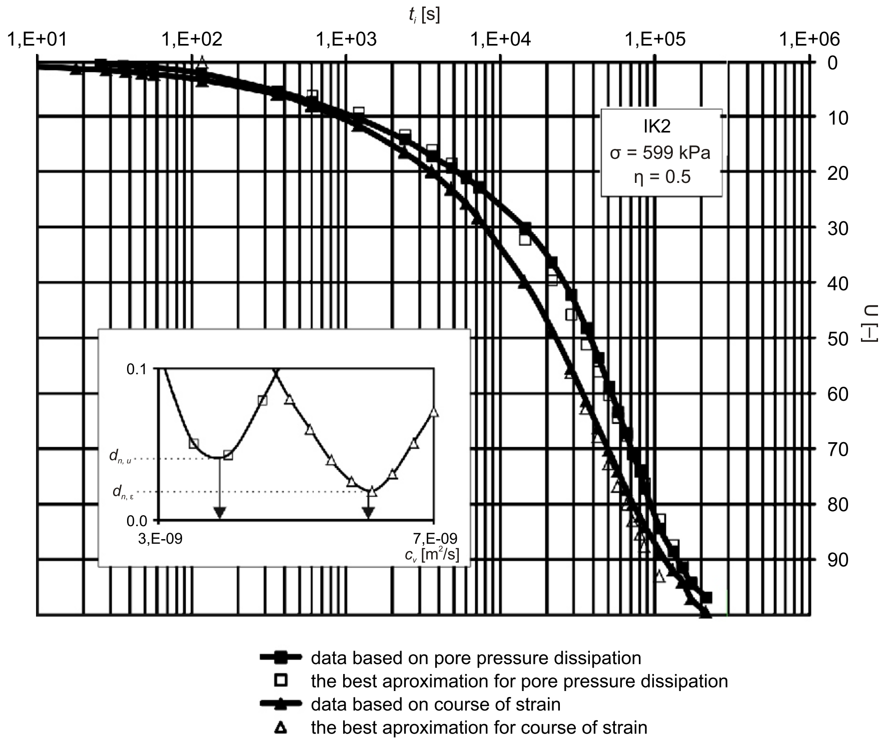


Fig. 3. Evaluation of optimal values of c_v by proposed method

at loads $_n$ = 900 and 1000 kPa, which is connected with a decreasing pore pressure contribution in loads transfer.

It is worth noticing that the discrepancies between the real behaviour of tested soil and theoretical consolidation solutions depend on the stage of test and on the way of the consolidation degree estimation, which is calculated on the basis of settlement progress path and/or pore pressure dissipation. The best adjustment of theoretical and experimental curve may be obtained by evaluation of weighted averages $d_{n,}$ and $d_{n,u}$ minimum values.

It is also worth mentioning that the $d_{n,}$ and $d_{n,u}$ values decrease at higher load levels, confirming higher reliability of soil behaviour description using theoretical models (Table 2).

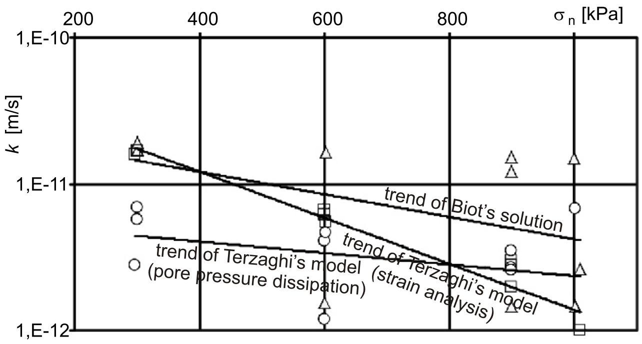
Simultaneously, the values $d_{n,u}$ are almost three times higher than $d_{n,}$. Thus, pore pressure as the more differentiated value, gives more diversified results of consolidation parameters, which are less consistent with the space phase of differential equations solution in consolidation theory.

Discussed relations and correlations are reflected in the set of results of permeability factor estimations on the basis of consolidation progress analysis, referring to Terzaghi's and Biot's theories (Fig. 4).

Table 2

Consolidation and permeability parameters obtained from interpretation of consolidation tests by approximation method on the basis of Terzaghi's theory

No. of sample	Effective stress	Initiale hight	Compressibility modulus	Parameters obtained from						factor
				course of strain			pore pressure u dissipation			
	$_n$	$_0$	M_0	$c_{v(\ )}$	d_n	$k_{(\ )}$	$c_{v(u)}$	$d_{n,u}$	$k_{(u)}$	
	[kPa]	[mm]	[kPa]	[m^2/s]	[–]	[m/s]	[m^2/s]	[–]	[m/s]	[–]
IK1	296	29.12	5100	8.0E-09	0.070	1.6E-11	1.5E-09	0.365	2.8E-12	4.5
	599	27.44	10000	6.5E-09	0.066	6.7E-12	1.2E-09	0.316	1.2E-12	4.4
	900	26.58	18500	5.1E-09	0.029	2.E-12	5.0E-09	0.049	2.7E-12	0.0
	1010	26.15	23500	2.4E-09	0.046	1.0E-12	1.8E-09	0.240	7.7E-13	0.3
IK2	299	29.01	4200	7.5E-09	0.058	1.7E-11	2.5E-09	0.167	5.8E-12	2.0
	599	26.99	9800	6.1E-09	0.019	6.2E-12	4.0E-09	0.043	4.1E-12	0.5
	900	26.16	21400	4.2E-09	0.052	2.0E-12	5.5E-09	0.054	2.6E-12	–0.2
	1001	25.79	25800	2.4E-09	0.085	9.3E-13	1.8E-08	0.111	6.8E-12	–0.9
IK3	299	29.28	5800	9.9E-09	0.081	1.7E-11	4.0E-09	0.091	6.9E-12	1.5
	601	27.76	12600	6.9E-09	0.025	5.5E-12	6.0E-09	0.032	4.7E-12	0.2
	900	27.10	20200	6.0E-09	0.033	3.0E-12	7.0E-09	0.060	3.5E-12	–0.1



Δ *k* according to Biot's solution
□ *k* according to course of strain – Terzaghi's model
○ *k* according dissipation of pore water pressure – Terzaghi's model

Fig. 4. Trends of decrease in permeability from tests' interpretation based on Biot's and Terzahgi's solutions

The obtained consolidation and permeability factors of tested soil decrease at subsequent load stages. The parameters obtained from the soil deformation analysis according to Biot's theory are higher than the parameters based on Terzaghi's solution calculated from the soil deformation analysis and pore pressure dissipation progress.

DISCUSSION

The obtained characteristics of consolidation-filtration parameters and their changes are also worth comparing with numerous modifications of classic Terzaghi's solution made throughout almost half of century. By their authors, these works are qualified as a non-linear solution of consolidation theory, however, they do not always lead to a strictly mathematical formulation of non-linearity.

The typical way of broadening the theory was through the successive abolishment of the initial simplifying assumptions. It also referred to the involvement of soil filtration parameters changeability.

However, most of the researchers tried to hold the basic value of the theory, which is expressing the solution in a function of non-dimensional time factor *T*, bonding in one numerical value soil filtration properties, compressibility characteristics and drainage path length.

Referring to the research program presented in the given paper, it is worth to consider the effects of solutions given in some of previous analysis and theoretical solutions.

Davis and Raymond (1965), referring to the logarithmic deformation model, assumed constant c_v, although *k* and m_v may change if the following condition is kept:

$$c_V = [k/(m_V g_W)] = \text{const} \qquad [23]$$

Supposing deformation u/ t is equal to the rate of water loss from the soil sample, they changed Terzaghi's equation to the form:

$$c_v \frac{\ ^2}{z^2} \quad \frac{u}{t} \qquad [24]$$

where: = $\log_{10}($ $_i'/$ $_f')$; $_f'/$ $_0'$ – final/initial effective stress in soil.

Disambiguation of their solution with Terzaghi's theory is presented as a set of *u*/ – *T* characteristics, which differ dependently on the value of $_f'/$ $_0'$ (1, 16) ratio. The research program presented in this paper assumes the values $_f'/$ $_0'$ < 1. Therefore, the effects of the delayed pore pressure dissipation, described by Davis and Raymond (1965), had a minor significance in the analysed research, especially on the mature stage of the process. Whereas incomplete mobilization of pore pressure, appearing under low increase of , was both observed both in the research on clay pastes led by Davis and Raymond (1965), as well as in the results described in the given article.

In turn, Barden and Berry (1965) assumed the model of filtration coefficient changeability, based on the e-log*k* relation, which is consistent with the quasi-linear model of changeability, obtained for the investigated clay pastes from Chmielów. The model described with the equations:

$$k/k_f \quad 1 \quad u/ \quad ^{B} \quad 1 \quad {}_k u^{N_B} \qquad [25]$$

relates the $_k$ = k_f/k_0 parameter with the earlier described $_f'/$ $_0'$ = $_B$ ratio, as well as N_B coefficient, which reflects the permeability in various cases.

Poskitt (1969) assumed that the filtration coefficient changeability can be expressed with the formula:

$$k \quad k_0 \frac{k_f}{k_0} \qquad [26]$$

where: $\frac{e_0 \quad e_i}{e_0 \quad e_f}$ and refers to the $U_{e,i}$ value described above.

To determine the m value, non-linear effects were included by adapting a perturbation method. The obtained solutions of consolidation progress in the function of non-linear time factor *T*, according to calculations of both Barden and Barry (1965), and Poskitt (1969), reveal that the consolidation progress characterized through pore pressure dissipation is delayed in relation to the characteristics based on deformation progress. This relation was confirmed in the analyzes of clays from Chmielów under the first load stage – therefore, the greatest participation of pore pressure and C_{IL0} value.

The non-linear consolidation theory, proposed by Gibson et al. (1967), assumed the linear changeability of consolidation ratio in the function of porosity changes. In the analysed tests such changeability is observed in the load range from 300 to 900 kPa. Relating to Gibson et al. (1967) results of model calculations on the clays from Chmielów, much greater changes of c_v coefficient were obtained with definitely smaller changes of porosity. It points to te existence of factors not included in the mentioned theoretical developments of filtration consolidation theory. Linear character of the $c_v = f(e)$ dependence and taking no account of compressibility in stress function are important limitations of the method. It complicates adaptation of the method for a range of practical tasks.

Butterfield and El-Bahey (1995) presented a method of broadening the range of consolidation progress calculations not only for the high load cases, but also for permeability changes of soil. The changed time parameter is necessary to be introduced into the classical Terzaghi's solution:

$$t' = (H/H_0)t \qquad [27]$$

As shown, it is necessary to apply the present values of decreasing drainage path length in this proposal.

This method allows for simplification of the calculations in comparison to the solution of Gibson et al. (1967), and obtaining the $c_v = f_1(1 + e)$ lub $c_v = f_2(\ ')$ consolidation ratio changeability characteristics.

Formerly introduced relations are also used in consolidation process analyses (Butterfield, 1979):

– transformation of soil volume changeability coefficient

$$m_v' = m_v(1 + e_0)/(1 + e) \quad [28]$$

– enhanced linearization of soil compressibility:

$$\ln[(1 + e)/(1 + e_0)] = -C_B \ln(\ '_f/\ _0') \quad [29]$$

Linearization of permeability changes in the consolidated soil, expressed as:

$$\ln(k/k_0) = A_B \ln[(1 + e)/(1 + e_0)] \quad [30]$$

for soils of various origins (Zindarcic et al.,1986), gives coefficient values A_B 6 under C_B 0.2. (Butterfield and El-Bahey, 1995).

When $C_B(A_B - 1) = 1$, simplification of basic continuity of hydrodynamical equations occurs, which in consequence allows for application of Terzaghi's solution with corrected determination of consolidation ratio:

$$c_v' = k/(\ _w m_v') = (k\ ')/(C_B\ _w) \quad [31]$$

Butterfield and El-Bahey (1995) compared the theoretical solutions given above with experimental consolidation progress of kaolinite clay in a large-scale oedometer. Calculations of the consolidation progress according to their proposal reveal a generally greater conformity with the results of analyses, than the classical solutions (Terzagi, 1925; Gibson et al., 1967). Applying time scaling as in [27] gives slightly increased U values at the initial stage of analysis and slightly decreased at the final stage. Even better conformity was obtained using solution with c_v' correction (according to [31]), where U calculation results are decreased only for the final stage of analysis. Then, in the interpretations analysed in the article, the optimized consolidation curve, in comparison to the experimental record, reveals decreased U values at the initial process stage and decreased at the final stage, where secondary consolidation effects may appear.

The comparison of the described methods with the experimental data allows for optimization of interpretation depending on the boundary conditions (loading program, drainage path length), as well as the character of the investigated soil. The methods proposed in the article allow for the initial assessment of consolidation (filtration) coefficient value with a lesser error, than the traditional point-based methods. Obtained characteristics can stand as a basis for determination of functional dependencies $\{c_v, k\} = f\{e/e_0,\ '/\ _0, H/H_0\}$, necessary for iterative selection of optimal, non-linear consolidation theory solutions.

CONCLUSIONS

1. Consolidation and permeability factor estimation, based on the proposed methodology, allows evaluating the character of filtration process and solid particles creeping, using newly introduced parameters: weighted averages of deviations $d_{n,}$, $d_{n,u}$ and factor.

2. Very low permeability of tested soils is reflected in both the values of estimated consolidation factors and pore pressure parameter C_{IL}. At higher load levels, the limitations of permeability factor are observed, which is caused by the decreasing conductivity of porous space.

3. Using three independent methods of calculation, similar tendencies in the trend of the decrease of permeability with decrease of loading are observed. Values k obtained from Biot's method on the basis of the sample settlement progress are higher and more diversified than the results obtained from Terzahgi's method both strain and pore pressure dissipation analysis. Taking into consideration the safety of insulating barriers, it is better to include methods that give higher values of permeability.

4. The observed sensitivity of consolidation behaviour in soil paste indicates the necessity of adjusting the research program to the construction and exploitation of artificial insulating soil barriers (thickness, consistency, expected loads). Further studies are recommended, among others, on the influence of structural ageing on consolidation-permeability characteristic changes.

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